Nucleon structure from $N_f = 2 + 1$ DWF simulations

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Outline

1. Introduction

2. Simulation parameters

3. Preliminary results

    - g_A/g_V
    - Moments of quark distributions
    - Form factors

4. Summary

1. Introduction

Motivation :

understand nucleon structure from first principle

We calculate matrix elements related to nucleon structure on $N_f = 2+1$ DWF configuration.

- g_A/g_V
  Well determined experimentally: $g_A/g_V = 1.2673(35)$
- Moments of quark distributions
  Deep inelastic scattering; structure functions
  $\langle x \rangle_q \rightarrow$ Unpolarized: $F_1(x, Q^2), F_2(x, Q^2)$
  $\langle x \rangle_{\Delta q} \rightarrow$ Polarized: $g_1(x, Q^2), g_2(x, Q^2)$
- Form factors
  Elastic scattering

$$
\begin{aligned}
F_1(q^2) &= \frac{1}{(1+q^2/M_V^2)^2}, \quad \langle r_{ch}^2 \rangle = 12/M_V^2, \\
G_A(q^2) &= \frac{g_A}{(1+q^2/M_A^2)^2}, \quad \langle r_{ax}^2 \rangle = 12/M_A^2
\end{aligned}
$$

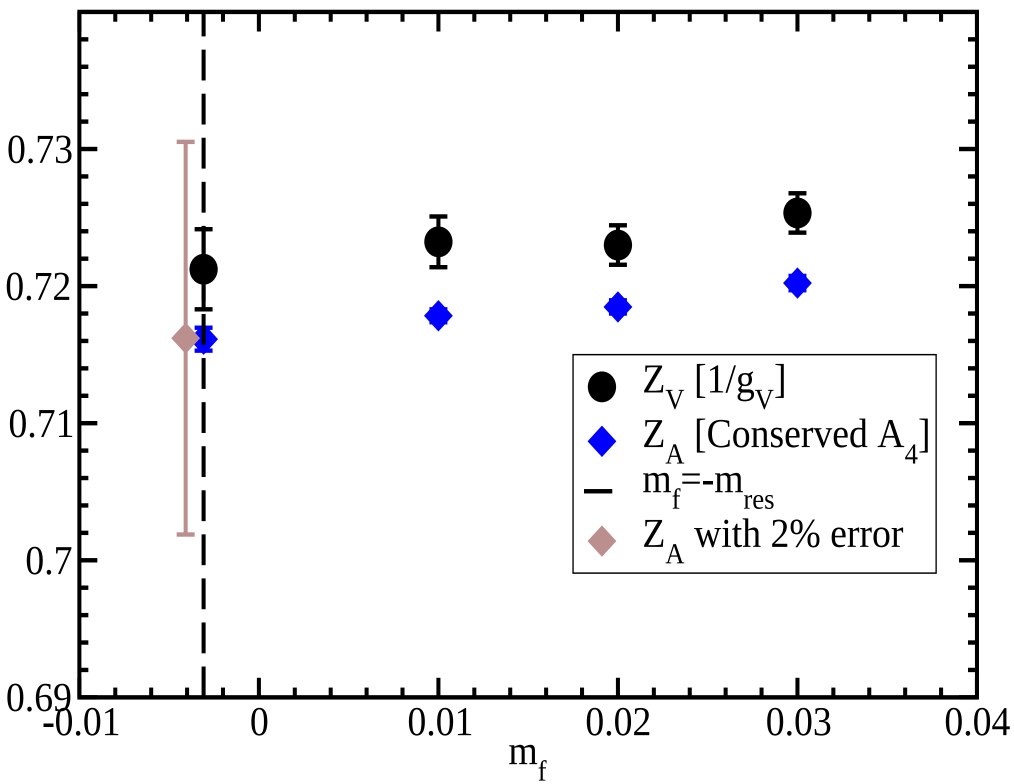
RBC-UKQCD generated $N_f = 2 + 1$ dynamical configuration.

1. u, d quark mass is as lower as $m_\pi = 310$ MeV.
   Investigation of nucleon structure in chiral regime

2. Chiral symmetry on lattice

3. Physical volume 3 fm

Chiral symmetry on lattice



If chiral symmetry is exact, $Z_V = Z_A$ at chiral limit.

Z_V is determined by Nucleon form factor

$$Z_V g_V^{lat} = F_1(0) = 1$$

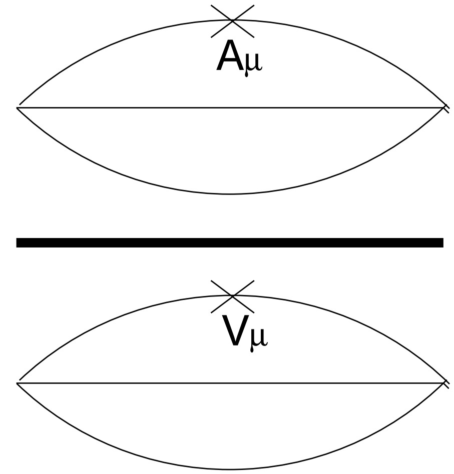
Z_A is determined by Conserved axial-vector current $\mathcal{A}_0$

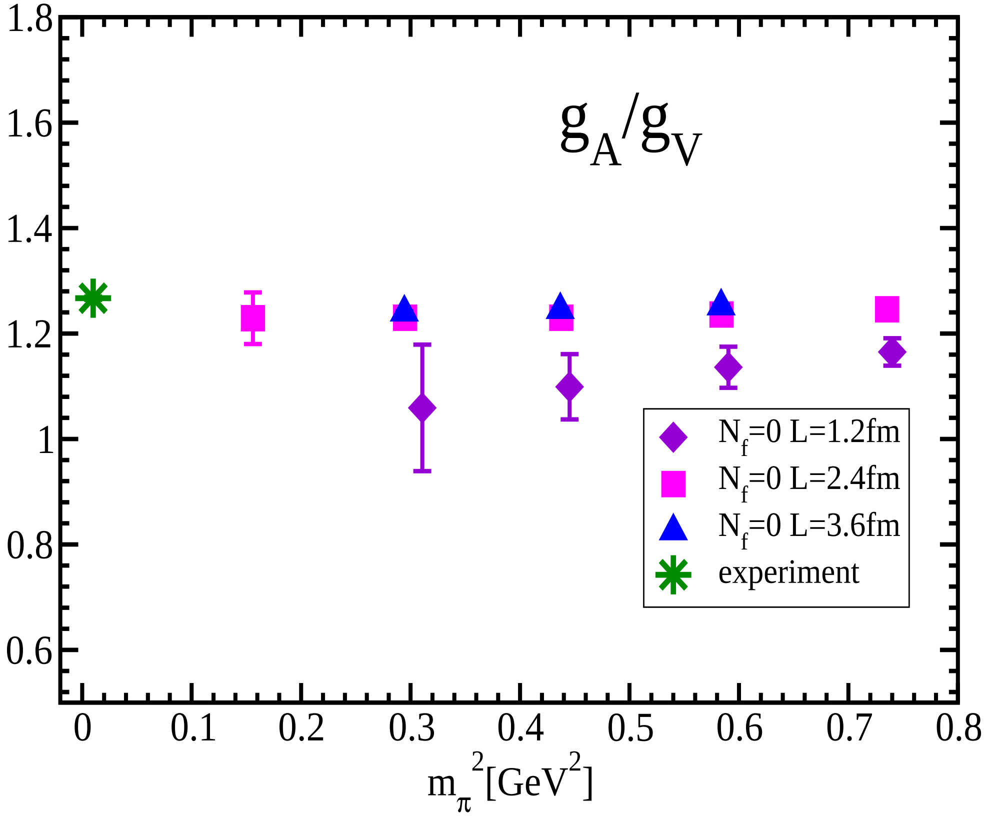
Z_A is consistent with Z_V within 2% at chiral limit.

Finite volume effect of nucleon matrix element

g_A/g_V is a simple, basic physical quantity of nucleon structure.

It is easy to calculate with DWF due to $Z_V/Z_A \approx 1$.





Large finite volume effect is seen in heavy m_π region.

g_A/g_V at 2.4 fm almost agrees with one at 3.6 fm.

$L \approx 2.5$ fm is enough for nucleon calculation.

RBC-UKQCD generated $N_f = 2 + 1$ dynamical configuration.

1. u, d quark mass is as lower as $m_\pi = 310$ MeV.
   Investigation of nucleon structure in chiral regime

2. Chiral symmetry on lattice
   $Z_V = Z_A$ is satisfied within a few %.

3. Physical volume 3 fm
   Volume is large enough for nucleon calculation, based on $N_f = 0$ calculation.

3. Simulation parameters

- $N_f = 2+1$ Iwasaki gauge + Domain Wall fermion actions
- $\beta = 2.13$ $a^{-1} = 1.62$ GeV $M_5 = 1.8$ $m_{\text{res}} = 0.003$
- Lattice size $24^3 \times 64 \times 16$ ($La \approx 3$ fm)
- $m_s = 0.04$ fixed (close to m_s^{phys})
- quark masses and confs.

m_f	m_π[MeV]	# of confs. $\times$ N_{meas}
0.005	310	52×4
0.01	390	119×4
0.02	520	49×4
0.03	690	53×4

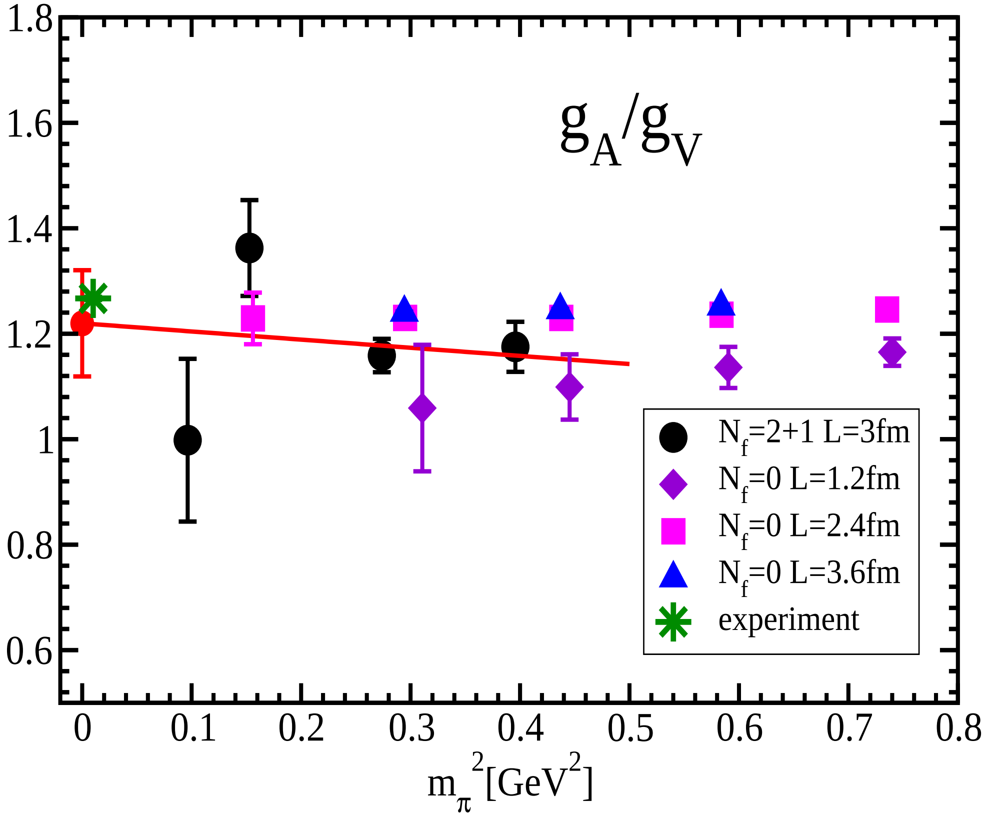
Results at two lighter masses do not have good accuracy, so that

all results are preliminary.

- We focus only on iso-vector quantities. (no disconnected diagram)

4. Preliminary results

4.1. g_A/g_V



m_π is lighter than m_π in $N_f = 0$ case.

Results at two lighter m_π has larger error and fluctuation.

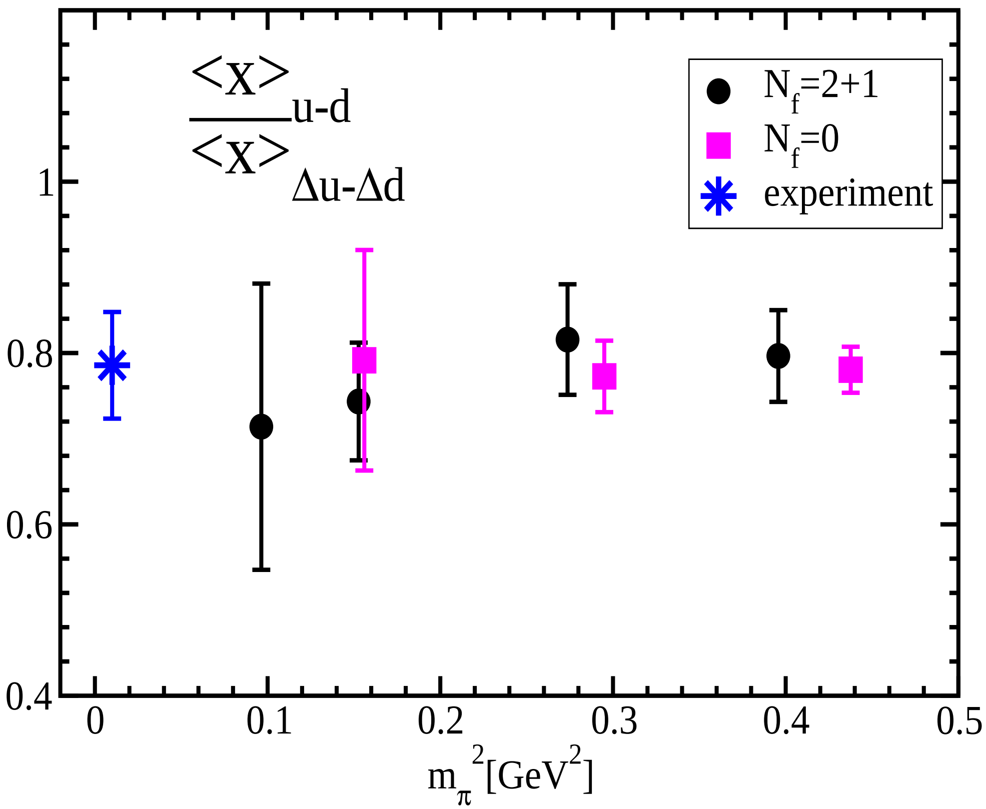
There seems to be no dynamical effect.

Preliminary result

$$g_A/g_V = \begin{cases} 1.22(10) & \text{(lat.)} \\ 1.267(4) & \text{(exp.)} \end{cases}$$

We will confirm the result is reliable by improving statistics.

4.2. Moments of quark distributions



Unpolarized : $\langle x \rangle_{u-d}$
Polarized : $\langle x \rangle_{\Delta u - \Delta d}$

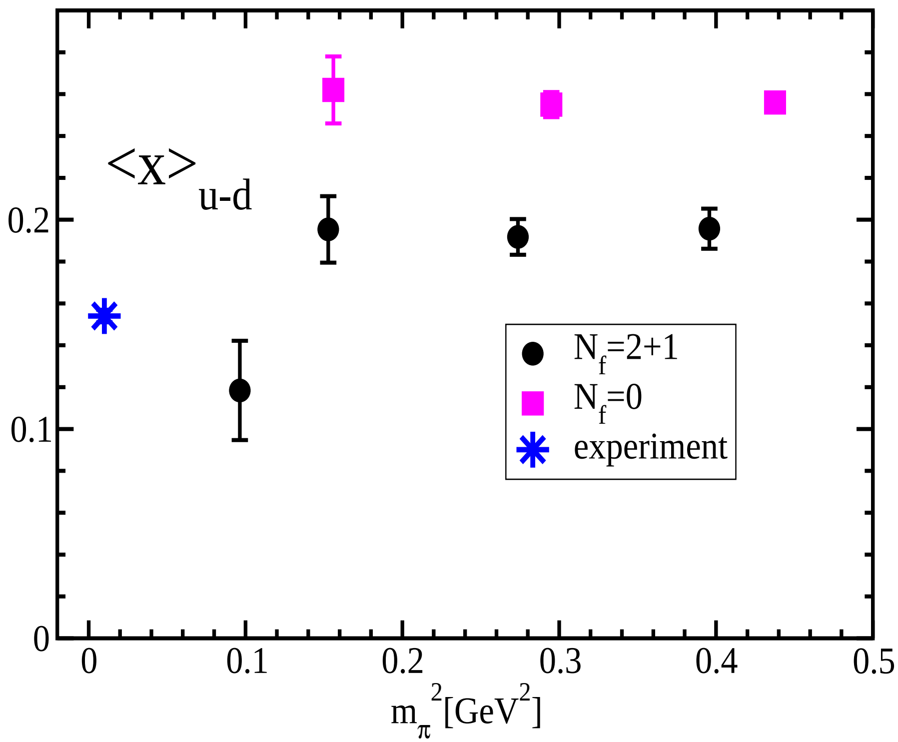
If chiral symmetry is exact, renormalizations of $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ are same.

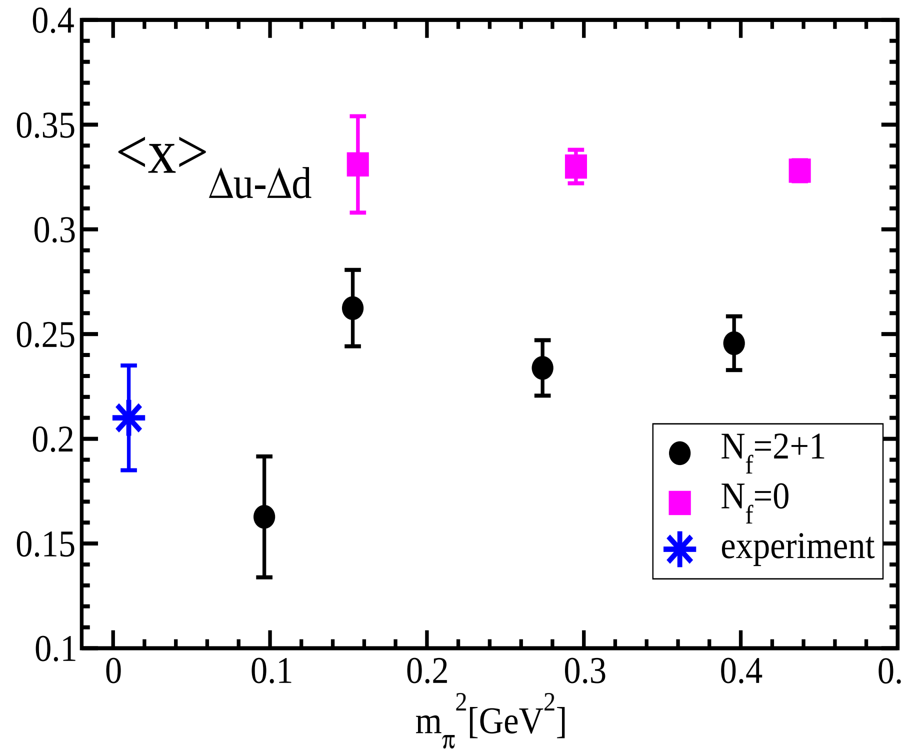
Result is consistent with experiment as well as in $N_f = 0$ case.

However, ...

4.2. Moments of quark distributions (cont'd)

Each component in $N_f = 0$ is independent of m_π.





$\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ are closer to experiment, and have some m_π dependence.

Perturbative $Z(\text{2GeV}) = 0.88(5)$ from PLB641,67

$\rightarrow$ We will calculate Z factor by non-perturbative method.

Preliminary result

4.3. Form factors

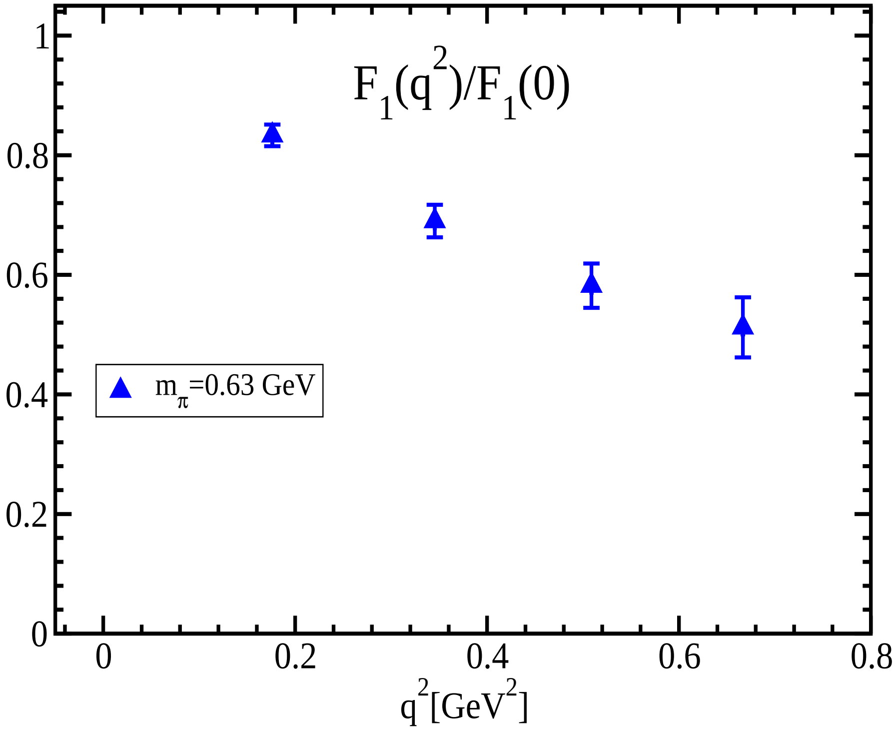
Iso-vector form factors $F_i^p - F_i^n$

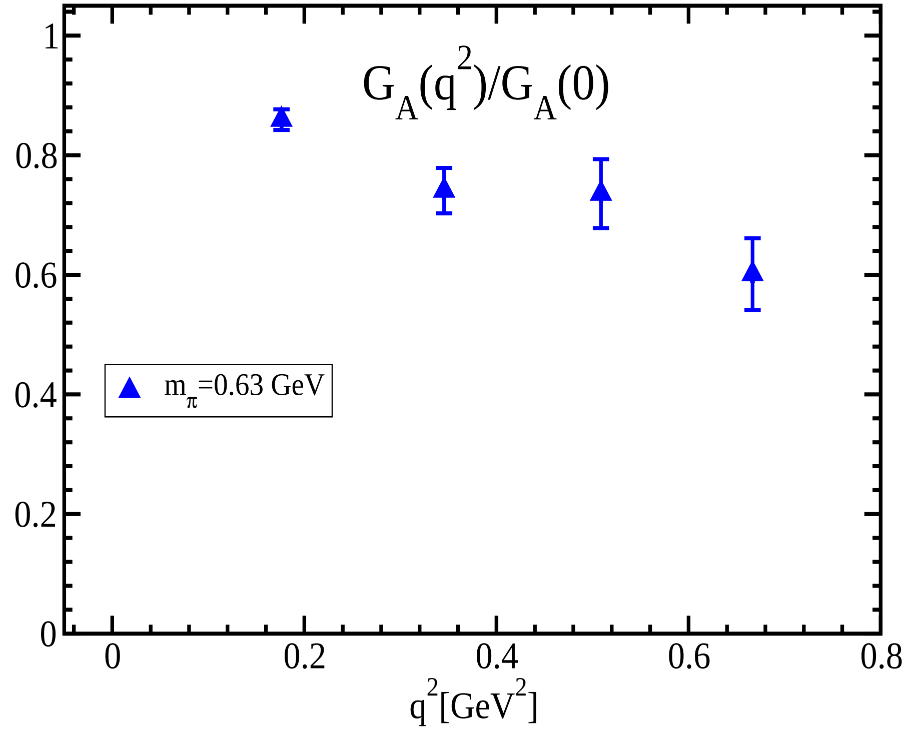
$$\begin{aligned}
\langle N, p|V_\mu(q)|N, p'\rangle &= \langle N, p|F_1(q^2)\gamma_\mu + i\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_N}|N, p'\rangle \\
\langle N, p|A_\mu(q)|N, p'\rangle &= \langle N, p|G_A(q^2)i\gamma_5\gamma_\mu + i\gamma_5 q_\mu G_P(q^2)|N, p'\rangle
\end{aligned}$$

$$q = p' - p,\ \left(\frac{Lp'}{2\pi}\right)^2 = 0, 1, 2, 3, 4$$

$$\begin{aligned}
F_1(q^2) &= \frac{1}{(1 + q^2/M_V^2)^2}, \quad M_V = 0.858(8)\ \text{GeV} \\
& \langle r_{ch}^2\rangle = 0.636(12)\ \text{fm} \\
G_A(q^2) &= \frac{g_A}{(1 + q^2/M_A^2)^2}, \quad M_A = 1.07(2)\ \text{GeV} \\
& \langle r_{ax}^2\rangle = 0.408(13)\ \text{fm}
\end{aligned}$$

4.3. Form factors (cont'd)





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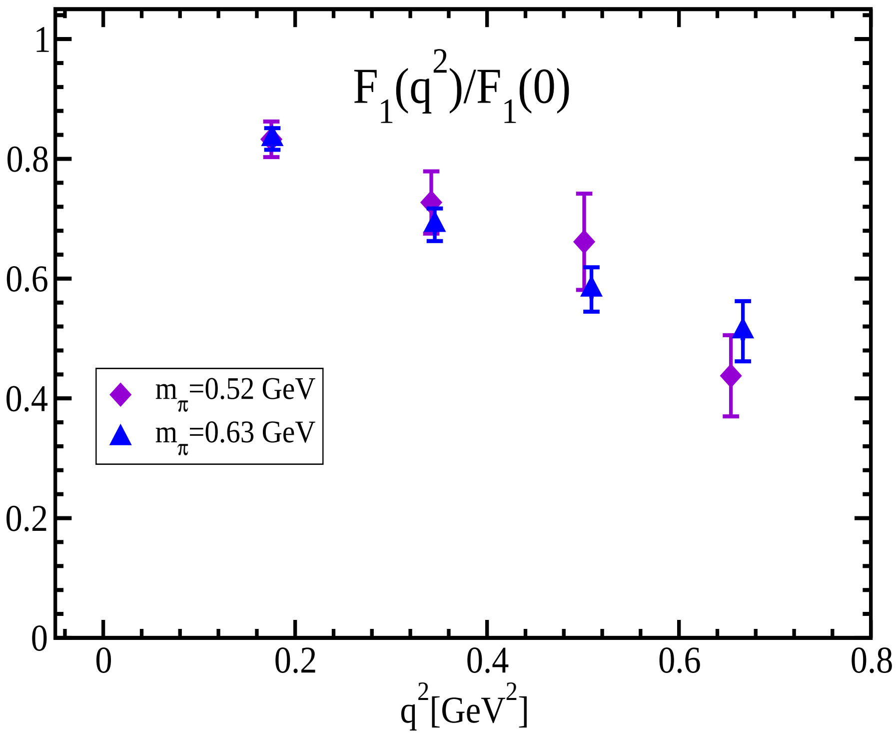

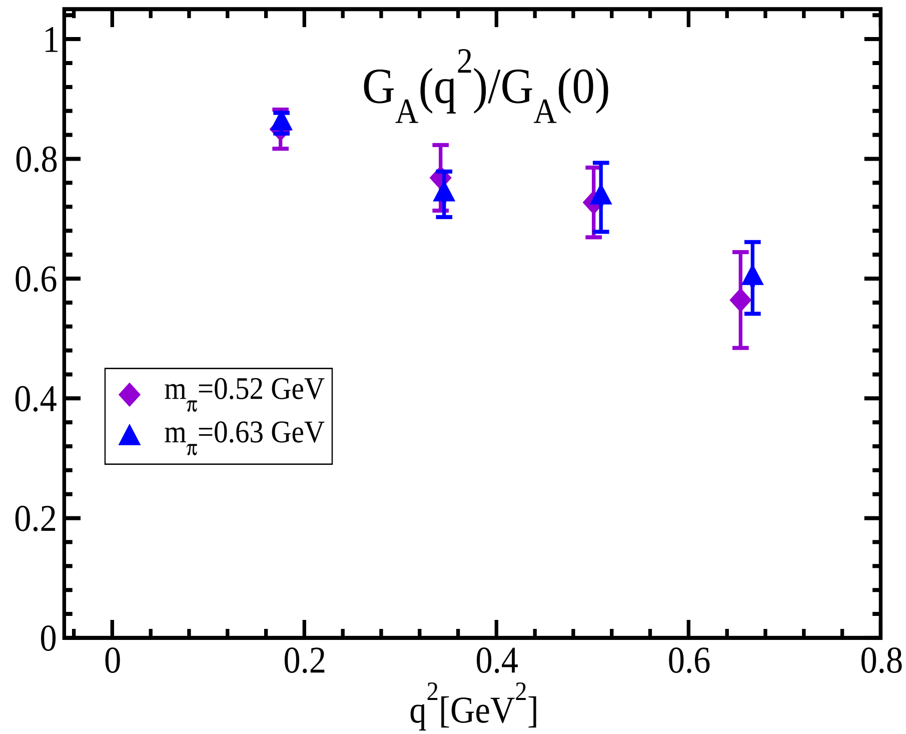

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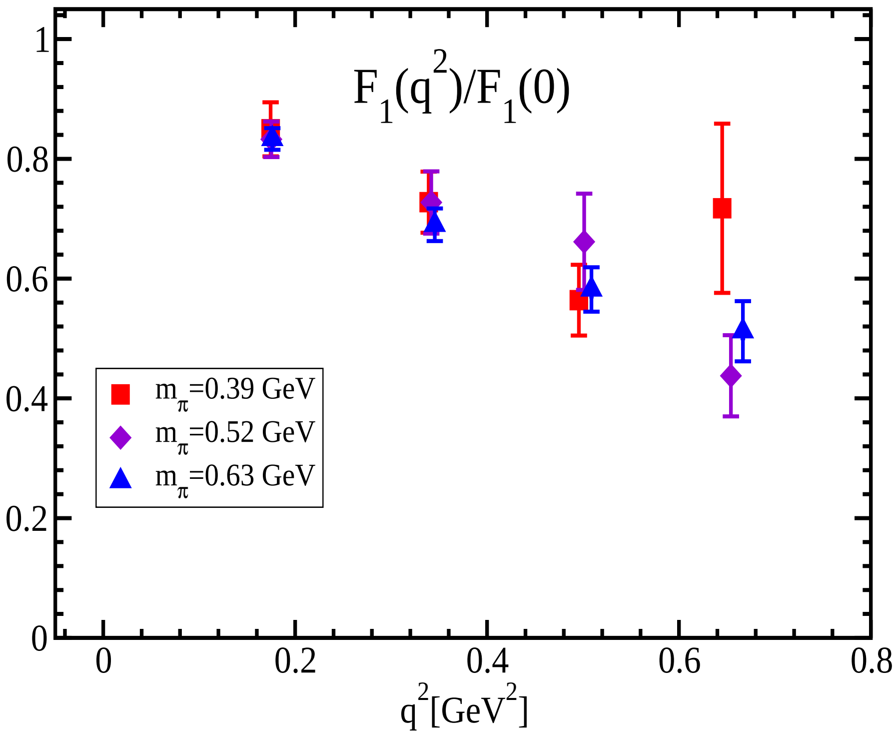

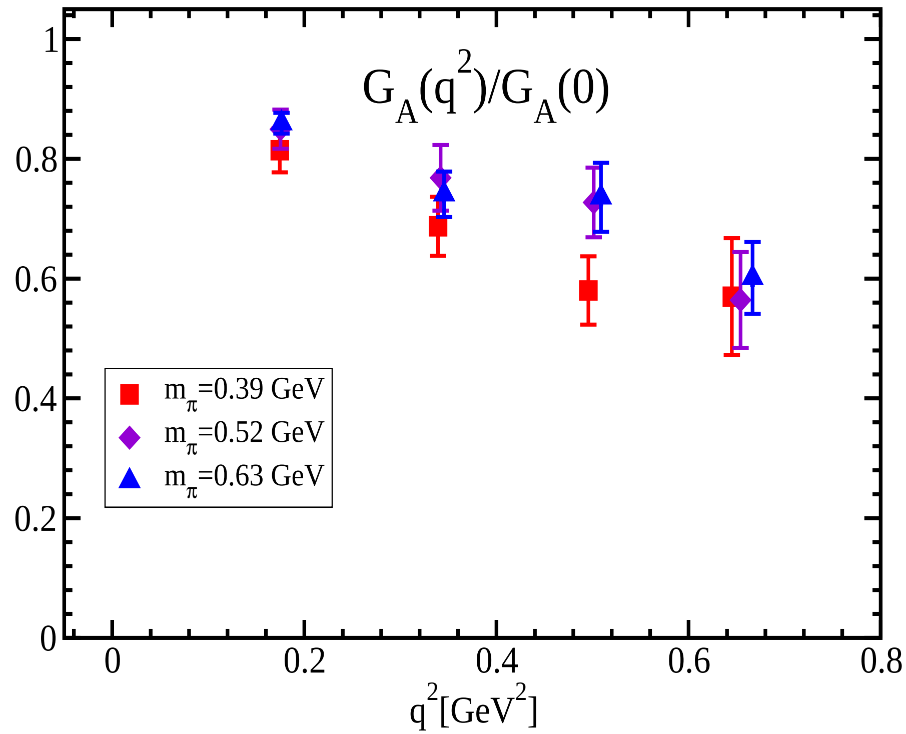

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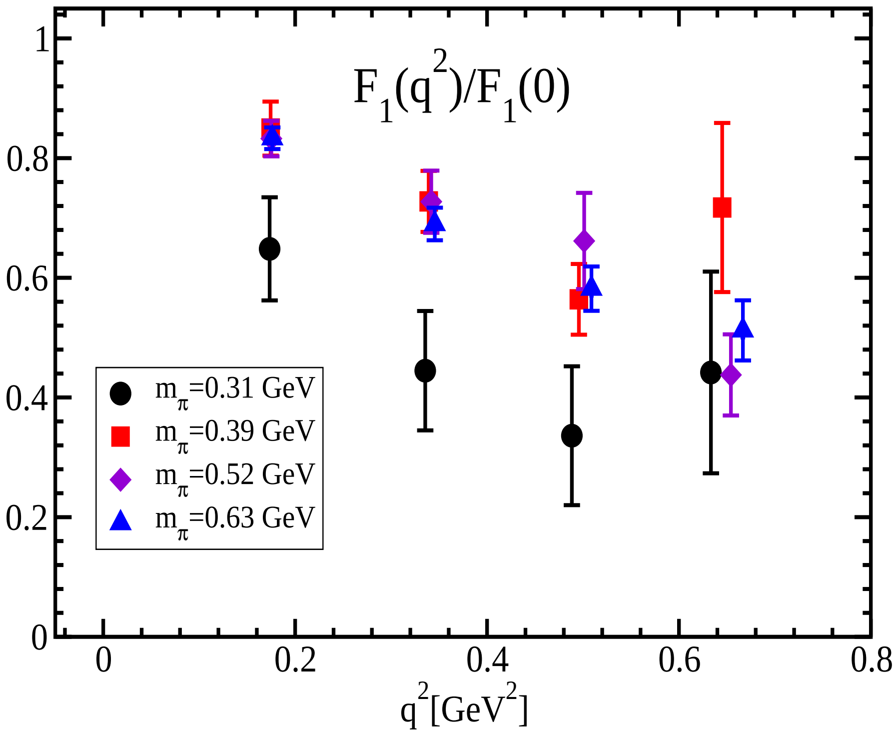
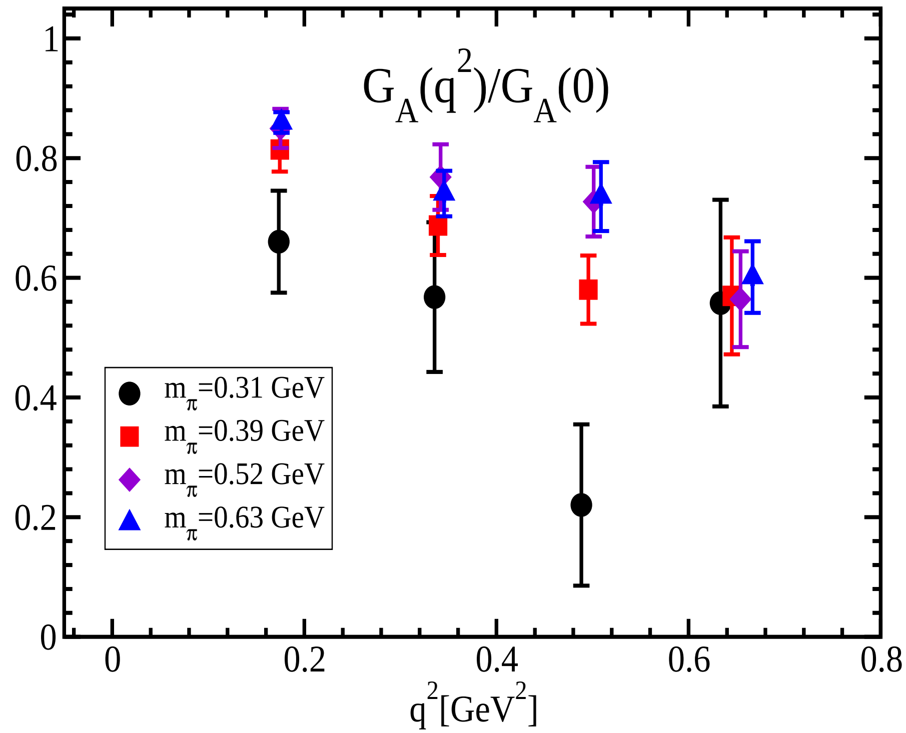
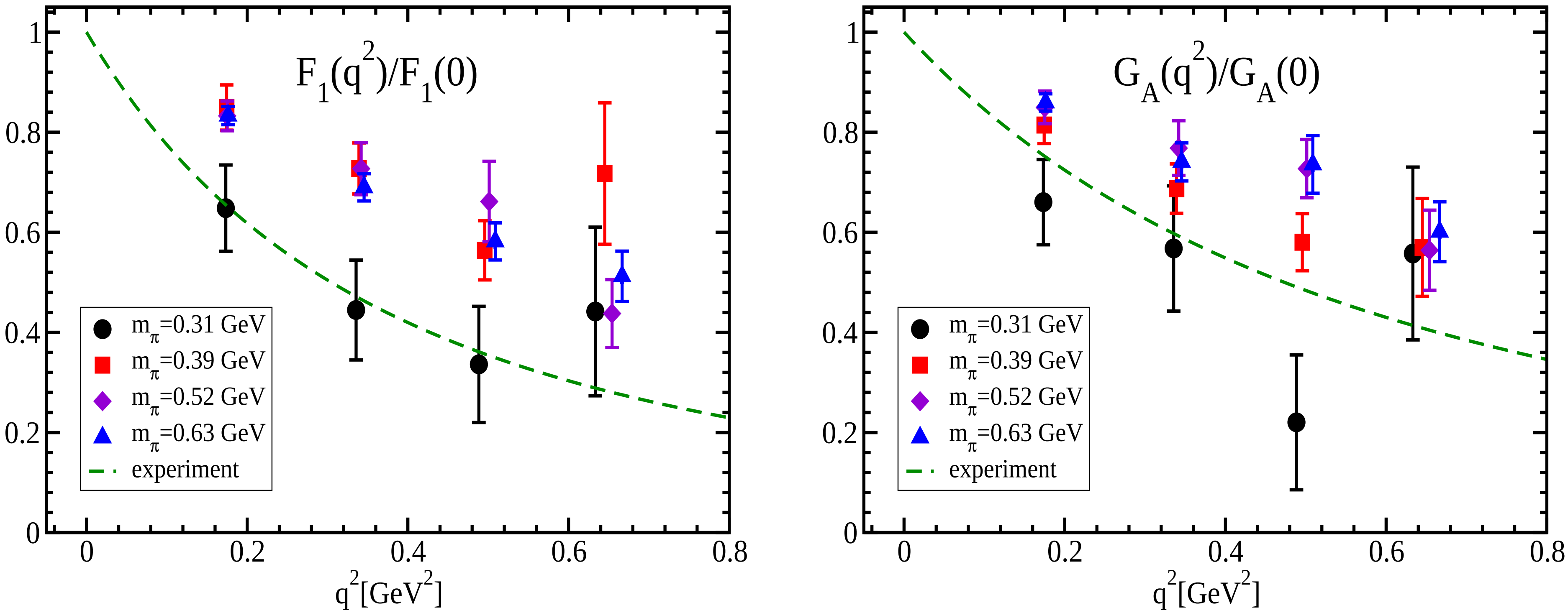




F_1 is almost independent of m_π except for lightest mass.

G_A has m_π dependence.

4.3. Form factors (cont'd)



F_1 is almost independent of m_π except for lightest mass.

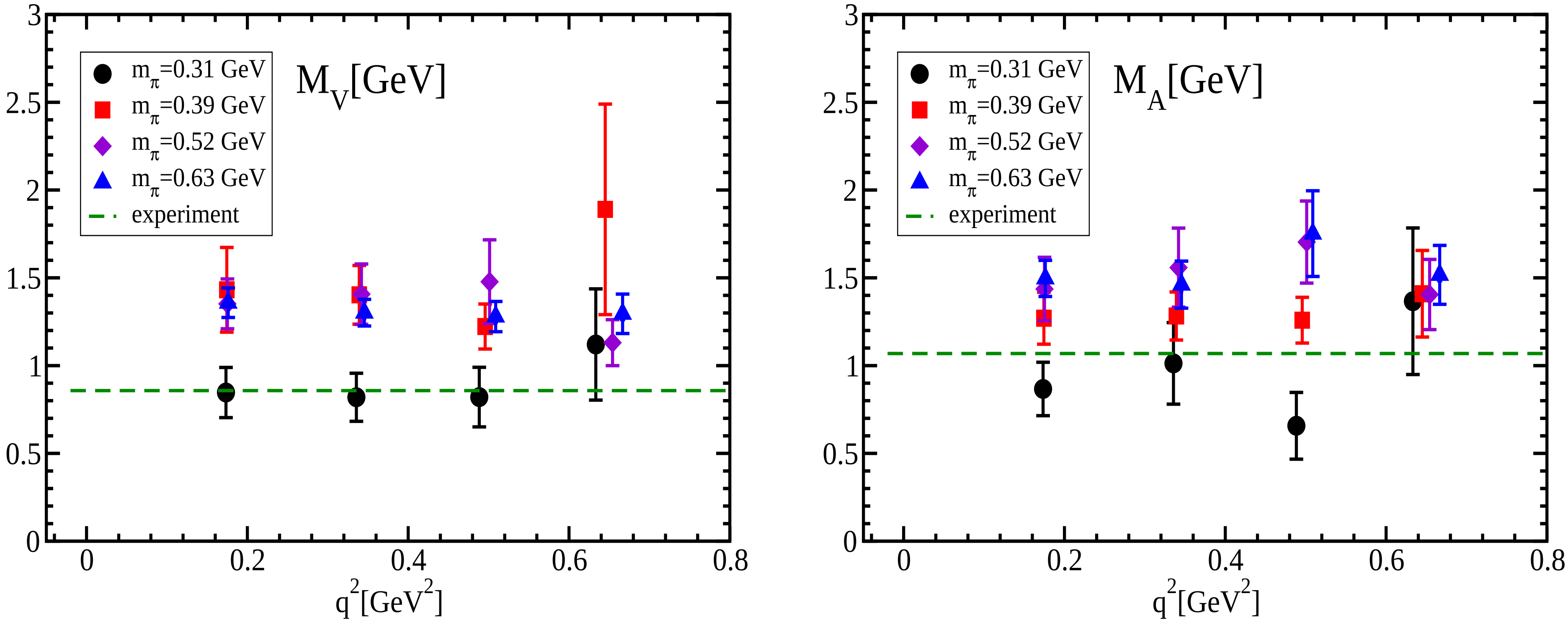
G_A has m_π dependence.

$$F_1(q^2) = \frac{1}{(1 + q^2/M_V^2)^2} \qquad G_A(q^2) = \frac{g_A}{(1 + q^2/M_A^2)^2}$$

4.3. Form factors (cont'd)

Effective M_V and M_A Preliminary result

$$M_V = \sqrt{\frac{\sqrt{F_1(q^2)} - 1}{q^2}}, \quad M_A = \sqrt{\frac{\sqrt{G_A(q^2)/g_A} - 1}{q^2}}$$

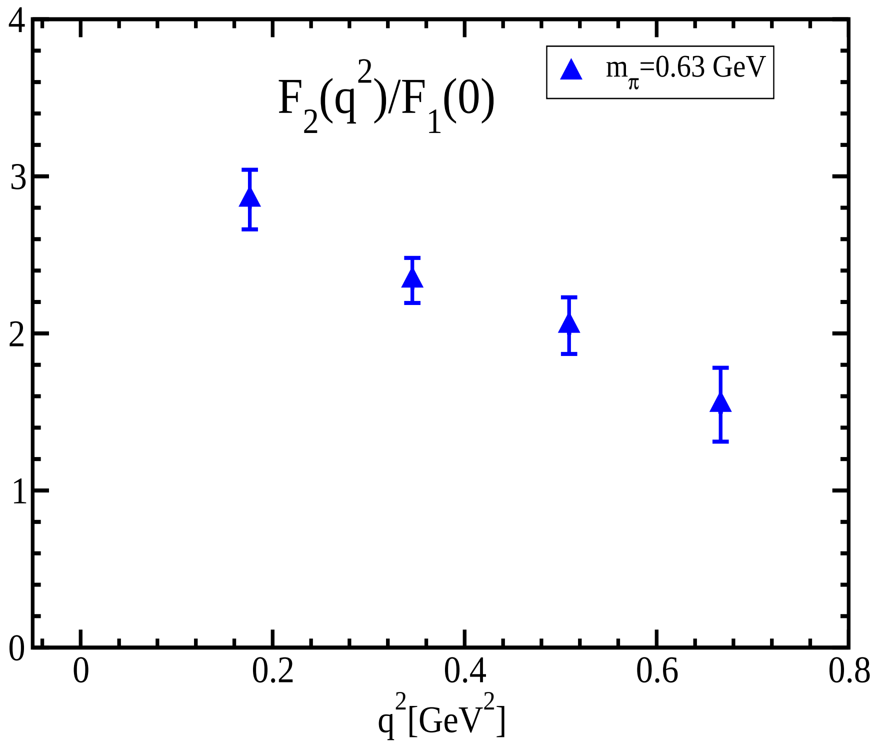


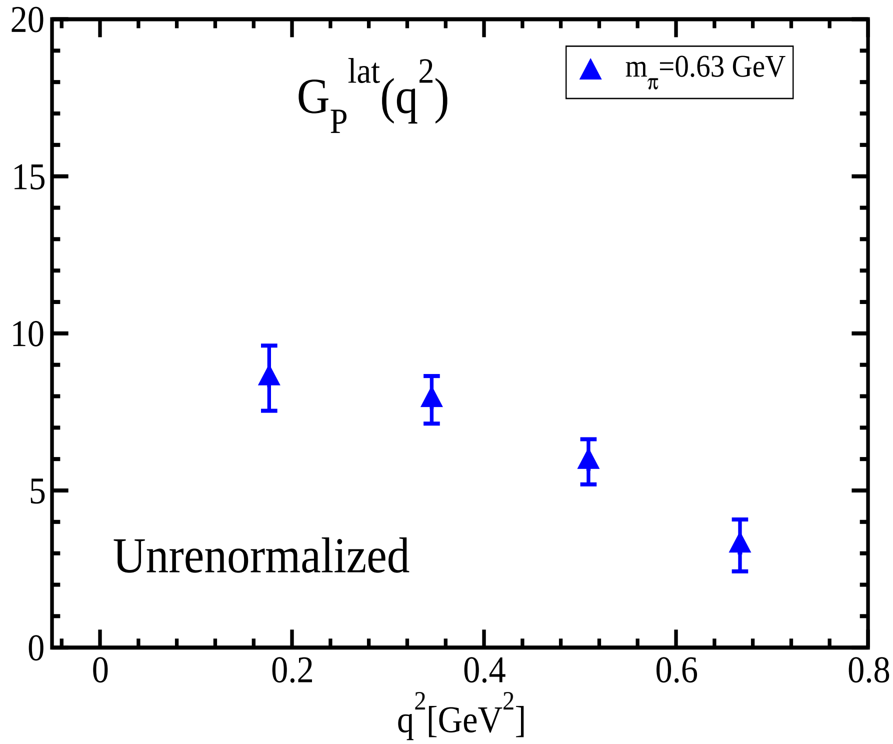
Effective M_V and M_A are reasonably flat.

M_V is almost independent of m_π except for lightest mass.

M_A has m_π dependence.

4.3. Form factors (cont'd)





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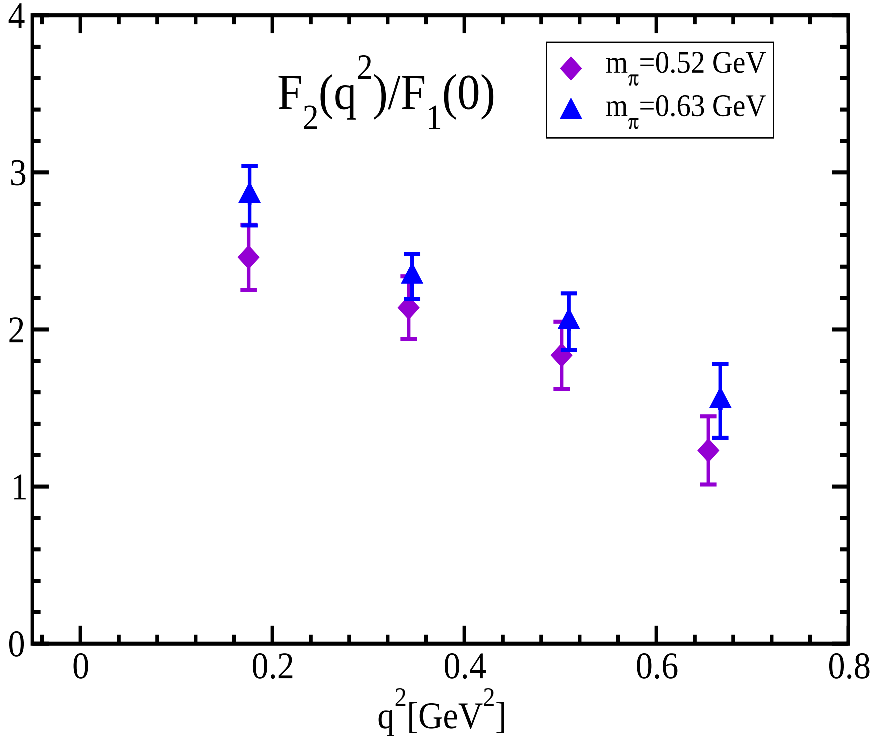

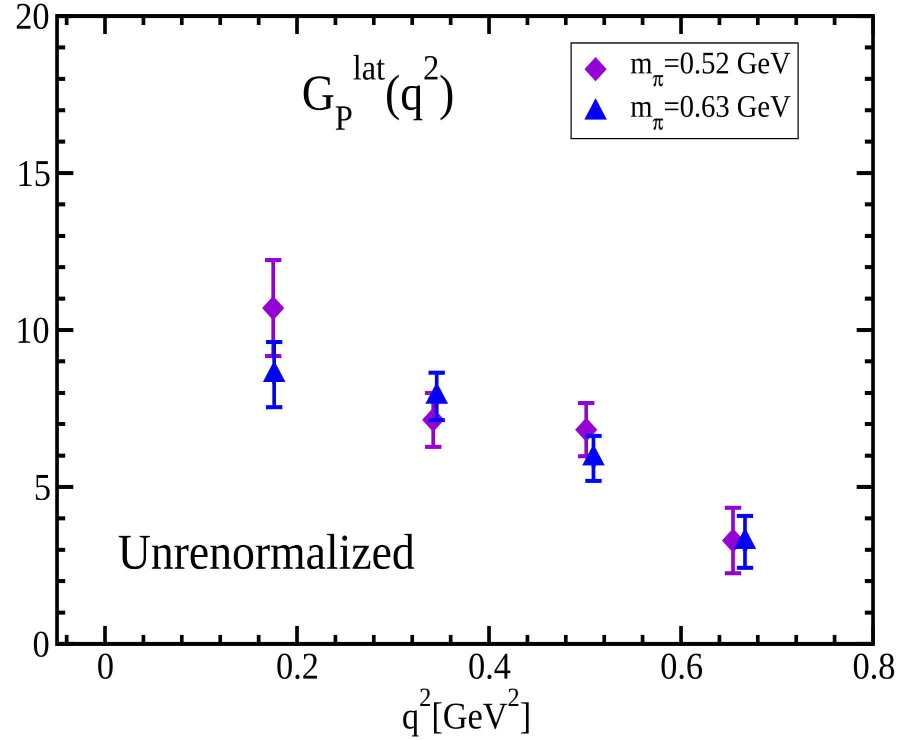

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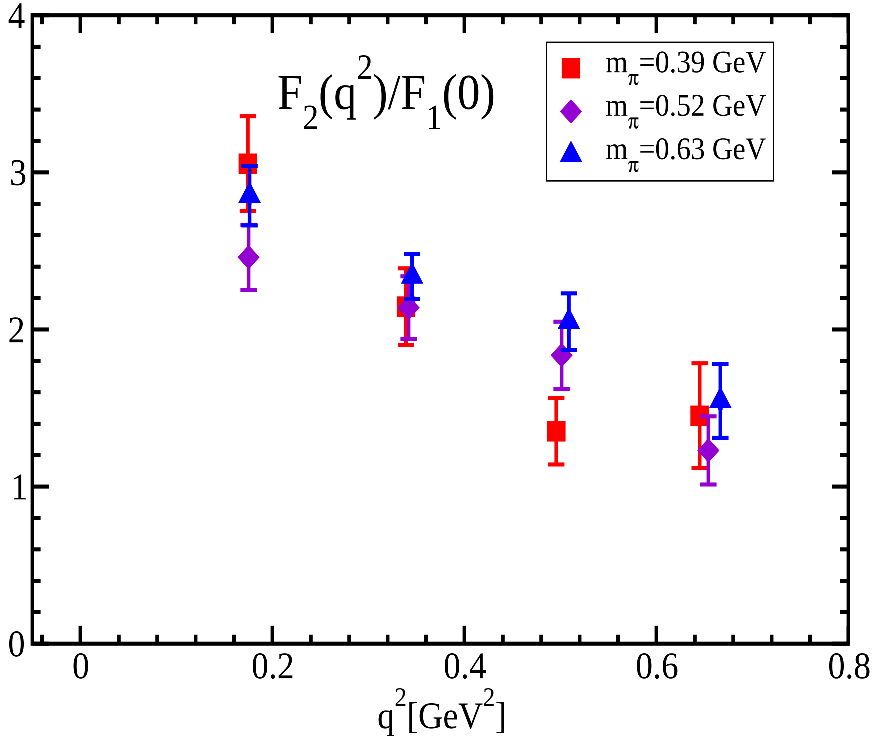

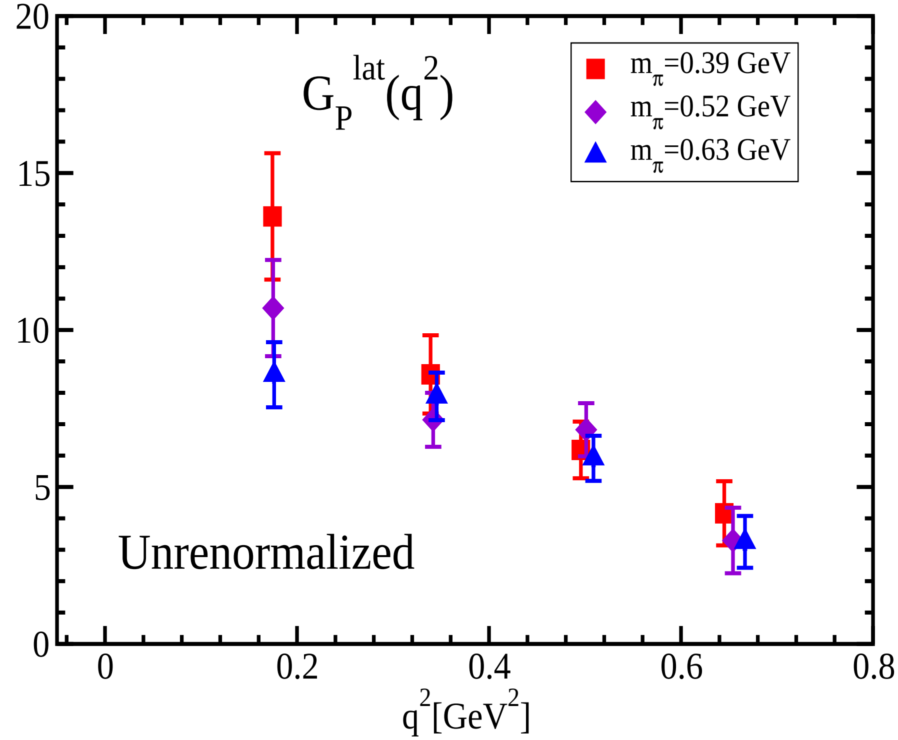

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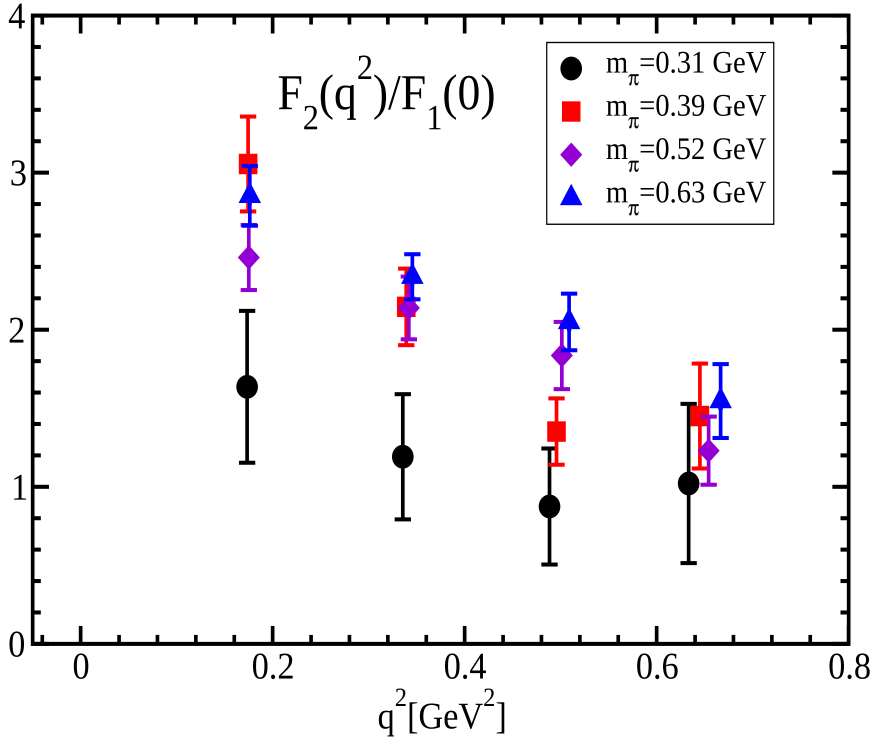
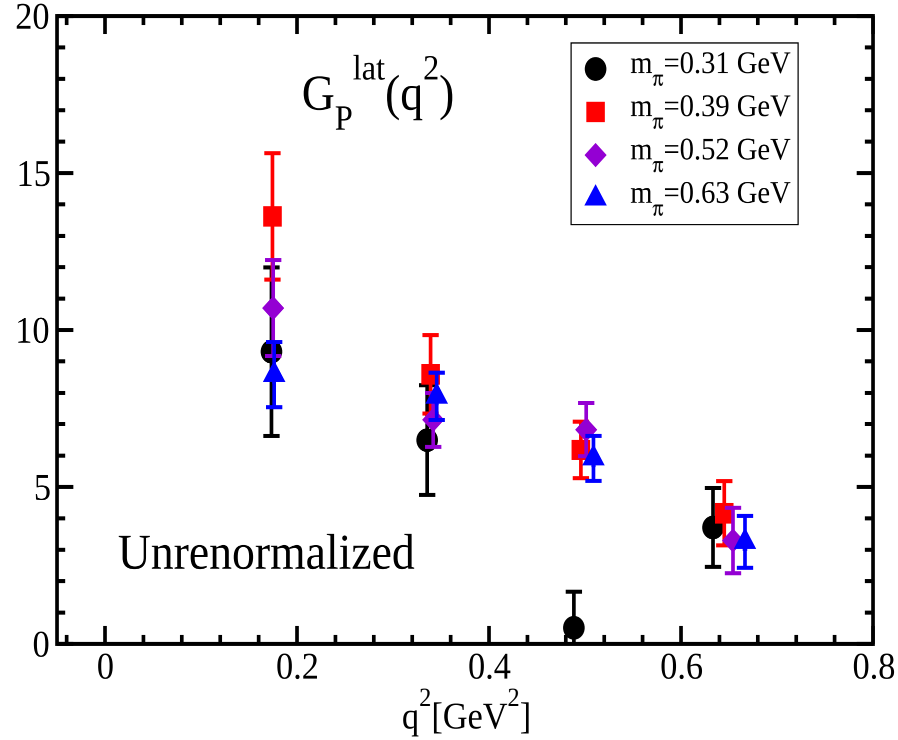


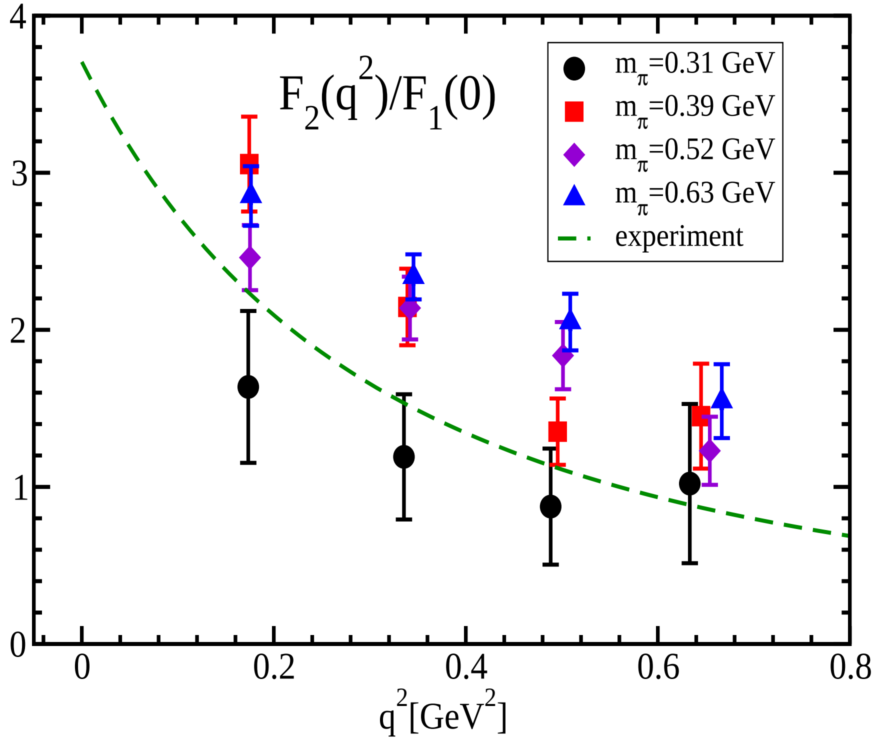


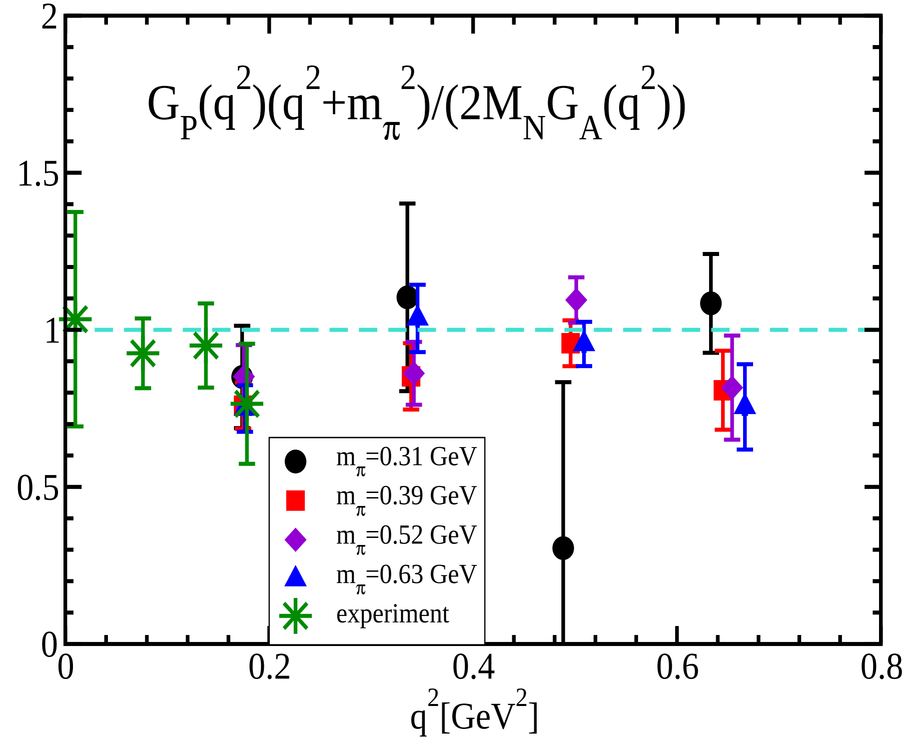
F_2 has m_π dependence, but one at lightest mass is much smaller.

G_P is explained by pion pole in current algebra,

$$G_P(q^2) = \frac{2M_N G_A(q^2)}{q^2 + m_\pi^2}.$$

4.3. Form factors (cont'd)





F_2 at lightest mass is below experiment.

Normalized G_P by pion pole and G_A is almost consistent with current algebra, and close to experiment.

$$G_P(q^2) = \frac{2M_N G_A(q^2)}{q^2 + m_\pi^2}$$

Preliminary result

5. Summary

- We calculated nucleon matrix elements with $N_f = 2 + 1$ dynamical domain wall fermions at light quark masses.
- All results are preliminary.
- We found encouraging and consistent results with experiments.

Future work

- We will improve statistics of $m_f = 0.005$ and $m_f = 0.01$ data.
- Next calculation is on larger lattice size and smaller lattice spacing.